Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Wednesday 8 January 2020

Morning (Time: 1 hour 30 minutes)	Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have: Mathematical Formulae and Statistical Tables (Lilac), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

: explanation
∵ is 'because'
∴ is 'therefore'

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

1. Find, in simplest form,

$$\int\left(\frac{8x^3}{3}-\frac{1}{2\sqrt{x}}-5\right)dx$$

(4)

integration method

① Write equation in easier form for integration.

$$\int \frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5\,dx = \int \frac{8}{3}x^3 - \left(\frac{1}{2}\times\frac{1}{\sqrt{x}}\right) - 5\,dx$$

$\because \frac{ab}{c} = \frac{a}{c}\times b$ or $a\times\frac{b}{c}$ & $\frac{1}{ab} = \frac{1}{a}\times\frac{1}{b}$

$$\int \frac{8}{3}x^3 - \left(\frac{1}{2}\times\frac{1}{x^{1/2}}\right) - 5\,dx$$

① indicies rule : $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$$\int \frac{8}{3}x^3 - \frac{1}{2}x^{-1/2} - 5\,dx$$

② indicies rule : $\frac{x}{a^b} = xa^{-b}$

② Integrate

$\because x^0 = 1$

$$\int \frac{8}{3}x^3 - \frac{1}{2}x^{-1/2} - 5x^0\,dx = \left[\left(\frac{8/3}{3+1}x^{3+1}\right) - \left(\frac{1/2}{-1/2+1}x^{-1/2+1}\right) - \left(\frac{5x^{0+1}}{0+1}\right)\right]$$

$$= \frac{2}{3}x^4 - x^{1/2} - 5x + C$$

← don't forget $+C$!!!
will lose a mark if not included

$\therefore \frac{2}{3}x^4 - x^{1/2} - 5x + C$

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Question 1 continued

Q1

(Total 4 marks)

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2. Given $y = 3^x$, express each of the following in terms of y. Write each expression in its simplest form.

(a) 3^{3x} **(1)**

(b) $\dfrac{1}{3^{x-2}}$ **(2)**

(c) $\dfrac{81}{9^{2-3x}}$ **(2)**

a) indicies rule : $a^{bc} = (a^b)^c = (a^c)^b$

$\Rightarrow \quad 3^{3x} = (3^x)^3 = (y)^3$

$\therefore \quad 3^{3x} = y^3$

b) $\dfrac{1}{3^{x-2}} = 3^{-(x-2)}$ ① $= 3^{2-x}$

① indicies rule $\dfrac{1}{a^b} = 1a^{-b}$

$3^{2-x} = 3^2 \div 3^x$ ② $= \dfrac{9}{3^x} = \dfrac{9}{(y)}$

② indicies rule $a^{b-c} = \dfrac{a^b}{a^c}$

$\therefore \quad \dfrac{1}{3^{x-2}} = \dfrac{9}{y}$

$9^2 = 81$

c) $\dfrac{81}{9^{2-3x}} = \dfrac{9^2}{9^{2-3x}} = 9^{2-(2-3x)}$ ② $= 9^{2-2+3x} = 9^{3x}$

② indicies rule $a^{b-c} = \dfrac{a^b}{a^c}$

$9^{3x} = (3^2)^{3x} = 3^{2\times 3x} = 3^{6x} = (3^x)^6 = (y)^6$

indicies rule : $a^{bc} = (a^b)^c = (a^c)^b$

$\therefore \quad \dfrac{81}{9^{2-3x}} = y^6$

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Question 2 continued

Q2

(Total 5 marks)

Same format as Q7 in Oct 2019 paper

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3.

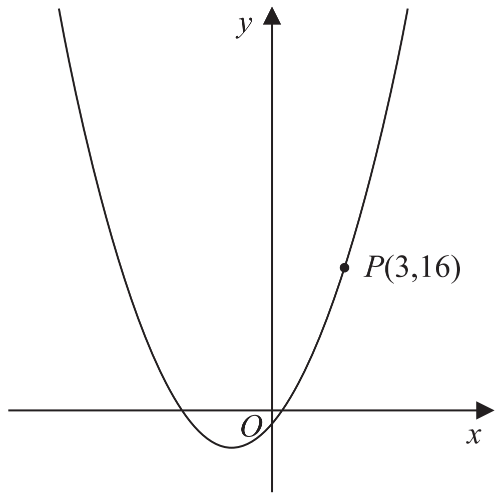

Figure 1

Figure 1 shows part of the curve with equation $y = x^2 + 3x - 2$

The point $P(3,16)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.
(2)

The point Q with x coordinate $3 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Write your answer in simplest form.
(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).
(1)

a) tangent means gradient of tangent is same as gradient of curve at point P

to find gradient of tangent, substitute x-value of P into dy/dx (the gradient function) of the curve.

$y = x^2 + 3x^1 - 2x^0$

$\therefore x^0 = 1$

$\frac{dy}{dx} = 2(x^{2-1}) + 1(3x^{1-1}) + 0(-2) = 2x + 3$

$\left.\frac{dy}{dx}\right|_{x=3} = 2(3) + 3 = 6 + 3 = 9$

$\therefore$ gradient $= 9$

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Question 3 continued

b) we will use formula for finding gradient: $m = \frac{y_1 - y_2}{x_1 - x_2}$

$P(3, 16)$ $Q((3-h), y_Q)$

① find y value of Q. As Q lies on curve C, substitute the x-coordinate of Q into equation of curve.

$y_Q = (3+h)^2 + 3(3+h) - 2 = (h^2+6h+9)+(9+3h)-2$
$= h^2 + 6h + 9 + 9 + 3h - 2 = h^2 + 9h + 16$

② find gradient with formula

$M_{PQ} = \frac{(h^2+9h+16) - 16}{(3+h) - 3} = \frac{h^2+9h}{h} = \frac{\cancel{h}(h+9)}{\cancel{h}} = h+9$

$\therefore M_{PQ} = h+9$

c) As $h \to 0$, gradient $PQ \to 9$, which is the same gradient as that of the tangent.

$(0)+9 = 9$

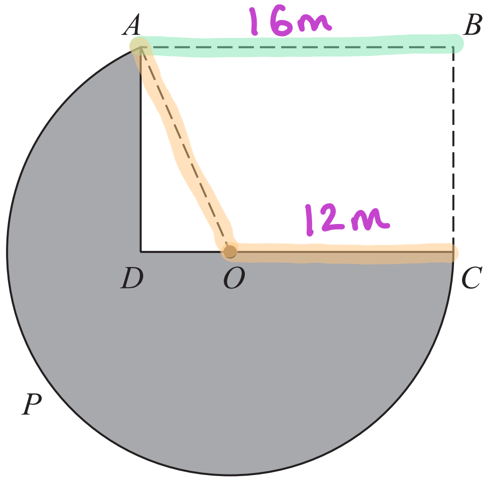
from part (a)

(Total 6 marks)

Q3

Turn over

4.



Not to scale

Figure 2

Figure 2 shows the plan view of a house *ABCD* and a lawn *APCDA*.

ABCD is a rectangle with $AB = 16$ m.

APCOA is a sector of a circle centre O with radius 12 m.

The point O lies on the line *DC*, as shown in Figure 2.

(a) Show that the size of angle *AOD* is 1.231 radians to 3 decimal places. **(2)**

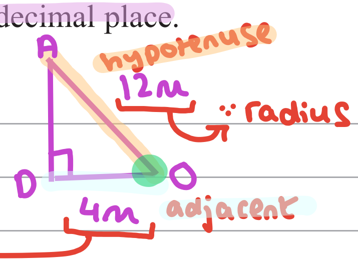
↑ check units!!

The lawn *APCDA* is shown shaded in Figure 2.

(b) Find the area of the lawn, in m^2, to one decimal place. **(4)**

(c) Find the perimeter of the lawn, in metres, to one decimal place. **(3)**

a) right angle triangle ADO



$AB = DC = 16m$

$DO + OC = 16m$

$DO + 12 = 16$

$\therefore DO = 4m$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \cos AOD = \dfrac{4}{12} = \dfrac{1}{3}$

$\angle AOD = \cos^{-1}\frac{1}{3} = 1.230959..$

$\therefore \angle AOD = 1.231$ (3dp)

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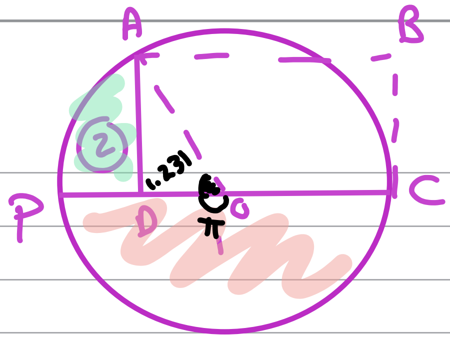
Question 4 continued

b) Split the shape to find area.



find area of 1 :

Area of a sector: $\frac{1}{2}r^2\theta$

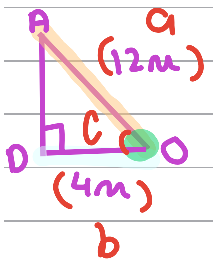
$\frac{1}{2}(12)^2(\pi) = 72\pi$ ← Sector POC

from part (a)

find area of 2 :

Area of sector AOP $= \frac{1}{2}(12)^2(1.231) = 88.632$

Area of triangle ADO $= \frac{1}{2}ab\sin C$



$\frac{1}{2}(12)(4)\sin(1.231) = 22.62774...$

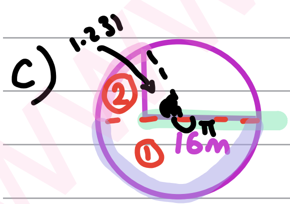
Area of section 2 = Sector − triangle
$= 88.632 - 22.6277...$
$= 66.004258...$

TOTAL AREA = 1 + 2

$72\pi + 66.0042... = 292.198...$

$\therefore$ Area APCDA $= 292.2\ m^2$ (1dp)

c)

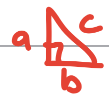


length of sector : $s = r\theta$

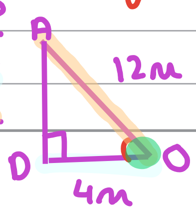
Sector 1 : $s = 12 \times \pi = 12\pi$

Sector 2 : $s = 12 \times 1.231 = 14.772$

find side AD. Triangle ADO is right angled triangle $\therefore$ use Pythagoras theorem : $a^2 + b^2 = c^2$



a = AD = ?
b = DO = 4
c = AO = 12



$AD^2 + 4^2 = 12^2$

-4^2 ($AD^2 = 12^2 - 4^2$) -4^2

square root ($AD = \sqrt{12^2 - 4^2}$) square root

$\therefore AD = 8\sqrt{2}$

Turn over ▶

P 6 0 7 9 6 A 0 9 2 8

Question 4 continued

Total perimeter = Sector ① + Sector ② + AD + DC

$= 12\pi + 14.772 + 8\sqrt{2} + 16$

$= 79.78482\ldots$

$\therefore$ Perimeter$_{APCDA}$ = 79.8 m (1dp)

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Question 4 continued

Q4

(Total 9 marks)

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5. (a) Find, using algebra, all solutions of

$$20x^3 - 50x^2 - 30x = 0$$

(3)

(b) Hence find all real solutions of

$$20(y+3)^{\frac{3}{2}} - 50(y+3) - 30(y+3)^{\frac{1}{2}} = 0$$

(4)

a) ① factorisation

$10(2x^3 - 5x^2 - 3x) = 0$

$10x\ (2x^2 - 5x - 3) = 0$

$10x\ (2x+1)(x-3) = 0$

② Solve for x:

$10x(2x+1)(x-3) = 0$

$10x = 0 \Rightarrow \therefore x_1 = 0$

$2x+1 = 0 \Rightarrow 2x = -1 \quad \therefore x_2 = -\frac{1}{2}$

$x - 3 = 0 \Rightarrow \therefore x_3 = 3$

$\therefore x = -\frac{1}{2}, 0, 3$

b) let $(y+3)^{\frac{1}{2}} = x$

$(y+3) = \left((y+3)^{\frac{1}{2}}\right)^2 = x^2$

$(y+3)^{3/2} = \left((y+3)^{1/2}\right)^3 = x^3$

$\therefore 20(y+3)^{\frac{3}{2}} - 50(y+3) - 30(y+3)^{1/2} = 20x^3 - 50x^2 - 30x = 0$

when $x_1 = 0$ ← Solutions from part (a)

$(y_1+3)^{1/2} = 0 \xRightarrow{\text{square}} y_1 + 3 = 0 \qquad \therefore y_1 = -3$

when $x_2 = -\frac{1}{2}$

$(y_2+3)^{1/2} \neq -\frac{1}{2}$ UNDEFINED $\because$ Cannot have a negative Square root of a number $\left(\sqrt{a} = a^{\frac{1}{2}}\right)$

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Question 5 continued

when $x_3 = 3$

$(y_3 + 3)^{\frac{1}{2}} = 3$ square $\Rightarrow$ $y_3 + 3 = 9$ -3

$y_3 = 6$ $\quad \therefore y_3 = 6$

$\therefore y = -3, 6$

Q5

(Total 7 marks)

Turn over

P 6 0 7 9 6 A 0 1 3 2 8

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6. The line l_1 has equation $3x - 4y + 20 = 0$

The line l_2 cuts the x-axis at $R(8, 0)$ and is parallel to l_1

(a) Find the equation of l_2, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

The line l_1 cuts the x-axis at P and the y-axis at Q.

Given that $PQRS$ is a parallelogram, find

(b) the area of $PQRS$,

(3)

(c) the coordinates of S.

(2)

a) Parallel lines have the same gradient(m) ∴ as $l_1 \parallel l_2$

$m_1 = m_2$

① rearrange l_1 equation in form $y = mx + c$ to find gradient

l_1: $3x - 4y + 20 = 0$

$+4y$: $3x + 20 = 4y$

$\div 4$: $\frac{3}{4}x + 5 = y$

$\therefore y = \frac{3}{4}x + 5$

gradient $= \frac{3}{4}$

② find equation of l_2 using line passing through (a, b) and gradient m

equation: $(y - b) = m(x - a)$

$a = 8$
$b = 0$
$m = \frac{3}{4}$

$(y - 0) = \frac{3}{4}(x - 8)$

③ rewrite in form $ax + by + c = 0$

$\times 4$: $y = \frac{3}{4}(x - 8)$

$4y = 3(x - 8)$

$-4y$: $4y = 3x - 24$

$0 = 3x - 4y - 24$

$\therefore 3x - 4y - 24 = 0$

$a = 3 \quad b = -4 \quad c = -24$

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Question 6 continued

b) ① Find coordinates of P & Q.

At P cutting x-axis ∴ y = 0

$l_1: 3x - 4y + 20 = 0$

$3x - 4(0) + 20 = 0$ $\quad -20$

$3x = -20$ $\quad \div 3$

$x = -\frac{20}{3}$ $\qquad P\left(-\frac{20}{3}, 0\right)$

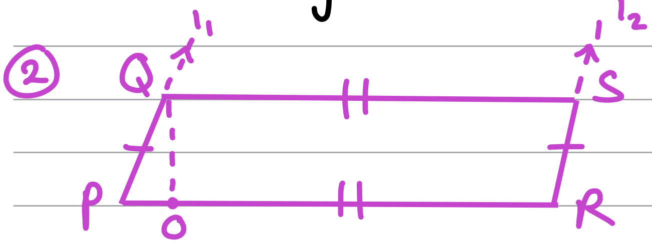
At Q cutting y-axis : x = 0

$3(0) - 4y + 20 = 0$ $\quad -20$

$-4y = -20$ $\quad \div -4$ $\qquad ∴ \quad Q(0, 5)$

$y = 5$

②



NOT ACCURATE
JUST TO HELP VISUALISE

Area of a parallelogram : $A = b \times h$

cannot have a negative length

base $= PR = \vec{PO} + \vec{OR} = \frac{20}{3} + 8 = \frac{44}{3}$

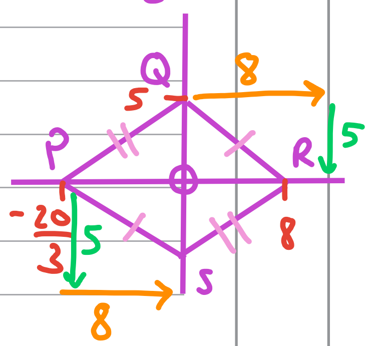
height $= QO = 5$

$A = bh = \frac{44}{3} \times 5 = \frac{220}{3}$

∴ Area $= \frac{220}{3}$ units^2

c) To get from Q(0,5) to R(8,0) : 8 units right, 5 units down. hence : $\vec{QR} \begin{pmatrix} 8 \\ -5 \end{pmatrix}$

$\vec{QR} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}$ so $\vec{PS}$ is $\begin{pmatrix} 8 \\ -5 \end{pmatrix}$



$P\left(-\frac{20}{3}, 0\right)$ $\quad ∴ \quad S\left(-\frac{20}{3} + 8, 0 - 5\right)$

$∴ \quad S\left(\frac{4}{3}, -5\right)$

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Question 6 continued

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Question 6 continued

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Q6

(Total 8 marks)

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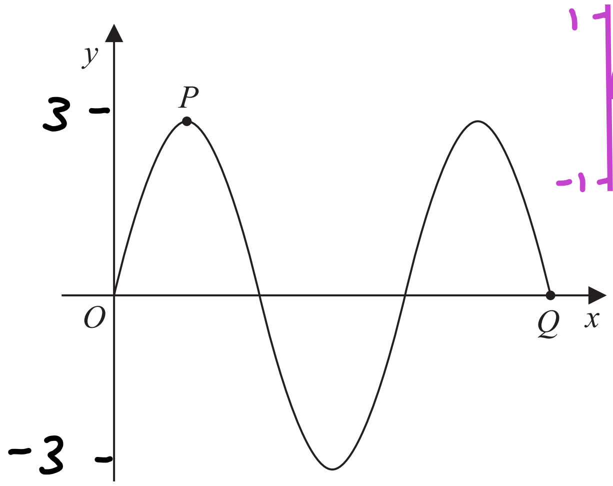
7.



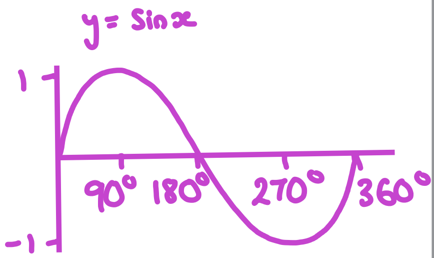


Figure 3

Figure 3 shows part of the curve C_1 with equation $y = 3\sin x$, where x is measured in degrees.

The point P and the point Q lie on C_1 and are shown in Figure 3.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

A different curve C_2 has equation $y = 3\sin x + k$, where k is a constant.

The curve C_2 has a maximum y value of 10

The point R is the minimum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

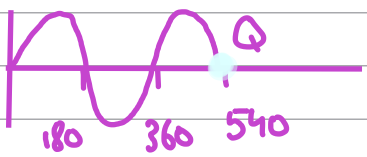
(2)

a) i) P is a maximum. $f(x) = \sin x \Rightarrow 3f(x) = 3\sin x$
$\therefore$ we multiply y-coordinate of $f(x)$ by 3 $\because$ vertical stretch by 3.

when $y = \sin x$, P is $(90°, 1)$
$\therefore$ when $y = 3\sin x$, P is $(90°, 1 \times 3)$

$P(90°, 3)$

ii) $y = \sin x$:



$Q(540°, 0)$

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P 6 0 7 9 6 A 0 1 8 2 8

for $y = 3\sin x$ $\quad Q(540°, 0 \times 3)$

$\therefore Q(540°, 0)$

b) $y = 3\sin x + k \Rightarrow y = 3f(x) + k$

Maximum is 10.

maximum of $3f(x)$ is y-coordinate 3.

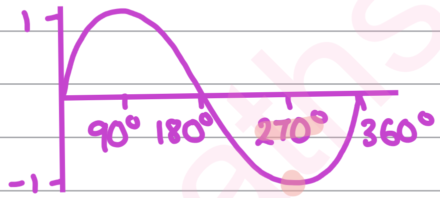
$\because$ K is outside f(x) bracket, we add K to f(x)

$\therefore y = 3f(x) + K = 10$

$3 + K = 10 \quad \therefore K = 7$

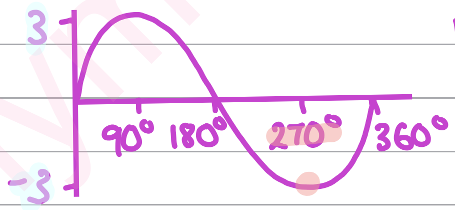
Minimum point R with smallest x-coordinate:

when $y = \sin x$



$R(270°, -1)$

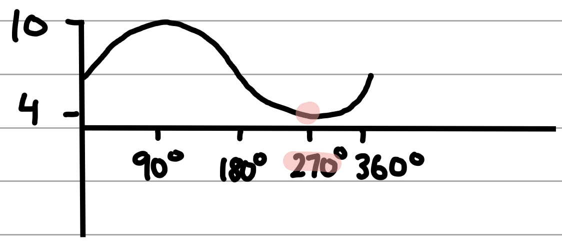
when $y = 3\sin x$



$R(270, -1 \times 3)$

$R(270, -3)$

$\therefore y = 3\sin x + K \longrightarrow y = 3\sin x + 7$



$R(270, 3(-1) + 7)$

$R(270°, 4)$

$\therefore R(270°, 4)$

Q7

(Total 5 marks)

8. The straight line l has equation $y = k(2x - 1)$, where k is a constant.

The curve C has equation $y = x^2 + 2x + 11$

Find the set of values of k for which l does not cross or touch C.

(6)

① equate the 2 equations together & simplify

$$x^2 + 2x + 11 = k(2x - 1)$$

$$x^2 + 2x + 11 = 2kx - k$$

$-2kx$, $+k$

$$x^2 + 2x + 11 - 2kx + k = 0$$

② group like-terms together:

$$x^2 + \underbrace{2x - 2kx} \; \underbrace{+11 + k} = 0$$

$$x^2 + (2 - 2k)x + (11 + k) = 0$$

③ l & C do not cross or touch ∴ they have no real root hence use discriminant to find range of k.

no real roots means that when the equation of the graph is $ax^2 + bx + c = 0$ discriminant is $b^2 - 4ac < 0$

$$\underset{a}{1}x^2 + \underset{b}{(2-2k)}x + \underset{c}{(11+k)} = 0$$

$$b^2 - 4ac < 0$$

$$(2-2k)^2 - 4(1)(11+k) < 0$$

④ Expand & Solve to find critical values

$$(2 - 2k)(2 - 2k) - 4(11 + k) < 0$$

$$(4k^2 - 8k + 4) - (44 + 4k) < 0$$

$$4k^2 - 8k + 4 - 44 - 4k < 0$$

Question 8 continued

$$4k^2 - 12k - 40 < 0$$

$\div 4$ $\qquad$ $\div 4$

$$k^2 - 3k - 10 < 0$$

$$(k+2)(k-5) < 0$$

when $k + 2 = 0$ $\quad$ $k = -2$

when $k - 5 = 0$ $\quad$ $k = 5$

Critical values are $k = -2, 5$

$$b^2 - 4ac < 0$$

$\therefore$ $k \neq -2$ & $k \neq 5$

when $k = (-2 - 1) = -3$

$(-3 + 2)(-3 - 5) = (-1)(-8) = 8$

$8 > 0$ but $b^2 - 4ac < 0$ $\therefore$ $-2 < k$

when $k = (5 + 1) = 6$

$(6 + 2)(6 - 5) = (8)(1) = 8$

$8 > 0$ but $b^2 - 4ac < 0$ $\therefore$ $k < 5$

NOT $\leq$

$\therefore$ $-2 < k < 5$

Q8

(Total 6 marks)

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x^2 + 9}{2\sqrt{x}} \qquad x > 0$$

Find the x coordinate of the point on the curve at which $\frac{dy}{dx} = 0$

(6)

① rewrite equation in form easier for differentiation

$$y = \frac{4x^2+9}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}(4x^2+9)$$

$$y = \frac{1}{2x^{\frac{1}{2}}}(4x^2+9)$$ ① indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$$y = \frac{1}{2}x^{-\frac{1}{2}}(4x^2+9)$$ ② indicies rule $\frac{a}{x^b} = ax^{-b}$

$$y = \frac{1}{2}\left(\left(x^{-\frac{1}{2}} \times 4x^2\right) + \left(x^{-\frac{1}{2}} + 9\right)\right)$$

$$= \frac{1}{2}\left(4x^{\frac{3}{2}} + 9x^{-\frac{1}{2}}\right) = \frac{4}{2}x^{\frac{3}{2}} + \frac{9}{2}x^{-\frac{1}{2}}$$

$$= 2x^{\frac{3}{2}} + 9x^{-\frac{1}{2}}$$

$$\therefore y = 2x^{\frac{3}{2}} + 9x^{-\frac{1}{2}}$$

② differentiate

$$\frac{dy}{dx} = \frac{3}{2}\left(2x^{\frac{3}{2}-1}\right) + \left(-\frac{1}{2}\right)\left(\frac{9}{2}x^{-\frac{1}{2}-1}\right)$$

$$= 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}}$$

③ find x-coordinate using $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}} = 0$$

Leave blank

DO NOT WRITE IN THIS AREA

P 6 0 7 9 6 A 0 2 2 2 8

Question 9 continued

$$3x^{1/2} - \frac{9}{4}x^{-3/2} = 0$$

$$x^{\frac{1}{2}}\left(3 - \frac{9}{4}x^{-2}\right) = 0$$

$$x^{-\frac{3}{2}} \div x^{\frac{1}{2}} = x^{-\frac{3}{2} - \frac{1}{2}} = x^{-\frac{4}{2}} = x^{-2}$$

indicies rule $\frac{a^b}{a^c} = a^b \div a^c = a^{b-c}$

$$x^{\frac{1}{2}}\left(3 - \frac{9}{4}x^{-2}\right) = 0$$

$x^{\frac{1}{2}} \neq 0 \quad \because \quad x > 0$

$$3 - \frac{9}{4x^2} = 0$$

$+\frac{9}{4x^2}$ $\quad$ $+\frac{9}{4x^2}$

$$3 = \frac{9}{4x^2}$$

$\times 4x^2$ $\quad$ $\times 4x^2$

$$12x^2 = 9$$

$\div 12$ $\quad$ $\div 12$

$$x^2 = \frac{9}{12}$$

$$x^2 = \frac{3}{4}$$

Square root $\quad$ Square root

$$x = \frac{\sqrt{3}}{\sqrt{4}}$$

$x \neq -\frac{\sqrt{3}}{2} \quad \because \quad x > 0$

$$\therefore \quad x = \frac{\sqrt{3}}{2}$$

Leave blank

Q9

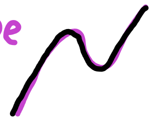
(Total 6 marks)

P 6 0 7 9 6 A 0 2 3 2 8

Turn over

Cubic graph $\because (x)(x^2)$

+ve x^3 shape



10. The curve C_1 has equation $y = f(x)$, where

Cross x-axis at $\boxed{\frac{3}{4}}$

$$f(x) = (4x - 3)(x - 5)^2$$

touch x-axis at $\boxed{x = 5}$

(a) Sketch C_1 showing the coordinates of any point where the curve touches or crosses the coordinate axes. Crosses y-axis at $x = 0$

$f(0) = (4(0)-3)(0-5)^2 = -75$ $\boxed{y = -75}$

(3)

(b) Hence or otherwise

(i) find the values of x for which $f\left(\frac{1}{4}x\right) = 0$

(ii) find the value of the constant p such that the curve with equation $y = f(x) + p$ passes through the origin.

(2)

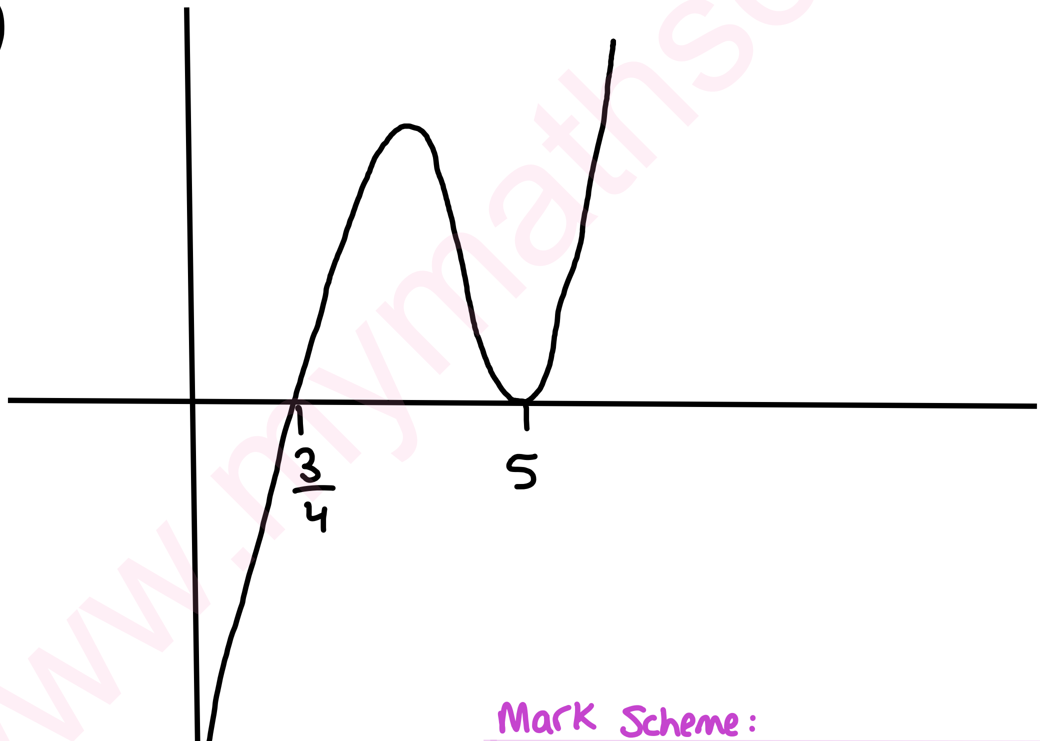
A second curve C_2 has equation $y = g(x)$, where $g(x) = f(x + 1)$

(c) (i) Find, in simplest form, $g(x)$. You may leave your answer in a factorised form.

(ii) Hence, or otherwise, find the y intercept of curve C_2

(3)

a)



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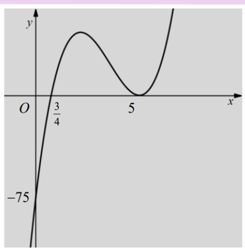


Shape for +ve x^3

Cuts x-axis at $\left(\frac{3}{4}, 0\right)$ and meets at $(5, 0)$

Crosses y-axis at $(0, -75)$

DO NOT WRITE IN THIS AREA

Question 10 continued

horizontal squash

b)i) $f\left(\frac{1}{4}x\right)=0$ inside $f(x)$ $\therefore$ affects x-value & inverse is done. x-value is divided by $\frac{1}{4}$ / multiplied by 4

$f(x)=0$, $x=5$ & $x=\frac{3}{4}$ from part (a)

so when $f\left(\frac{1}{4}x\right)=0$

$x = 5\times 4 = 20$ $\quad\therefore\quad$ $x = 3, 20$

$x = \frac{3}{4}\times 4 = 3$

ii) $y=f(x)+p$ outside $f(x)$ $\therefore$ affects y-coordinates.

translation, moving $f(x)$ by $\begin{pmatrix}0\\p\end{pmatrix}$ p units up.

y-intercept of $f(x)$ is -75 at $x=0$ $\therefore$ move up by 75 units to pass through origin $(0,0)$.

$\therefore\ p=75$

c)i) $g(x) = f(x+1)$

$g(x) = (4(x+1)-3)((x+1)-5)^2 = (4x+4-3)(x+1-5)^2$

$\therefore\ g(x) = (4x+1)(x-4)^2$ no need to expand

ii) y-intercept is when $x=0$

$g(0) = (4(0)+1)((0)-4)^2 = (1)(-4)^2 = 1(16) = 16$

$\therefore$ y-intercept of $g(x)$ is 16

Q10

(Total 8 marks)

P 6 0 7 9 6 A 0 2 5 2 8

11. A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}''(x) = \frac{6}{\sqrt{x^3}} + x \qquad x > 0$$

The point $P(4, -50)$ lies on the curve.

Given that $\mathrm{f}'(x) = -4$ at P,

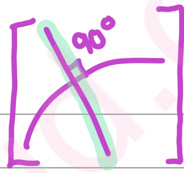
(a) find the equation of the normal at P, writing your answer in the form $y = mx + c$, where m and c are constants,

(3)

(b) find $\mathrm{f}(x)$.

(8)

a) normal is Perpendicular to Curve



∴ we find gradient of normal (m_n) using formula $M_{normal} \times M_{curve} = -1$

① find gradient of curve at point $P(4, -50)$

This is already given to us with the differential (gradient function) $f'(x) = -4$

② gradient of normal

$\div -4$ ($m_n \times -4 = -1$) $\div -4$

$m_n = \frac{-1}{-4}$

$m_n = \frac{1}{4}$

③ find equation of tangent using line passing through (a, b) and gradient M

equation: $(y - b) = M(x - a)$

$a = 4$

$b = -50$

$M = 1/4$

$(y - (-50)) = 1/4\,(x - 4)$

Question 11 continued

③ write in the form $y = mx + c$

$y + 50 = \frac{1}{4}(x-4) \Rightarrow y + 50 = \frac{1}{4}x - 1$

$\therefore y = \frac{1}{4}x - 51$

b) $f''(x) \xrightarrow{\text{integrate}} f'(x) \xrightarrow{\text{integrate}} f(x)$

$f''(x) \xleftarrow{\text{differentiate}} f'(x) \xleftarrow{\text{differentiate}} f(x)$

$\boxed{1}$ $f''(x)$ to $f'(x)$

① write $f''(x)$ in easier form for integration

$f''(x) = \frac{6}{\sqrt{x^3}} + x$

$f''(x) = \frac{6}{x^{\frac{3}{2}}} + x$ ① indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$f''(x) = 6x^{-\frac{3}{2}} + x^1$ ② indicies rule $\frac{a}{x^b} = ax^{-b}$

② integrate

$\int f''(x)\,dx = \int 6x^{-\frac{3}{2}} + 1x^1\,dx = \left[\frac{6}{-\frac{3}{2}+1} x^{-\frac{3}{2}+1} + \frac{1}{1+1} x^{1+1}\right]$

$= -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 + k$

③ find constant K

$f(x) = -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 + k$

when $x = 4$ at P, $f'(x) = -4$

$f'(4) = -12(4)^{-1/2} + \frac{1}{2}(4)^2 + k = -4$

$2 + K = -4$ $\quad -2$

$K = -6$ $\quad -2$

$\therefore f'(x) = -12x^{-1/2} + \frac{1}{2}x^2 - 6$

Turn over

P 6 0 7 9 6 A 0 2 7 2 8

Question 11 continued

$\boxed{2}$ $f'(x)$ to $f(x)$

① integrate

$\because x^0 = 1$

$$\int f'(x)\,dx = \int -12x^{-1/2} + \frac{1}{2}x^2 - 6x^0 \,dx$$

$$= \left[\left(\frac{-12}{-1/2+1}x^{-1/2+1}\right) + \left(\frac{1/2}{2+1}x^{2+1}\right) - \left(\frac{6}{0+1}x^{0+1}\right)\right]$$

$$= -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + c$$

② find constant c. for $P(4,-50)$ when $x=4$, $f(x)=-50$

$$f(4) = -24(4)^{1/2} + \frac{1}{6}(4)^3 - 6(4) + c = -50$$

$$= \frac{-184}{3} + c = -50$$

$$\frac{-184}{3} + c = -50$$

$+\frac{184}{3}$ $\qquad$ $+\frac{184}{3}$

$$c = \frac{34}{3}$$

$$\therefore f(x) = -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + \frac{34}{3}$$

Q11

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

P 6 0 7 9 6 A 0 2 8 2 8